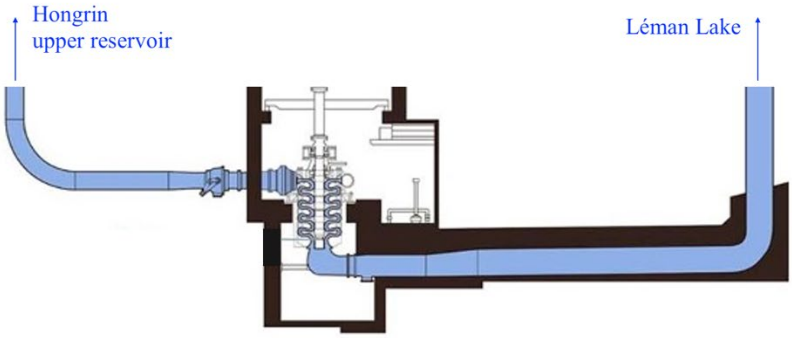
HYDRAULIC TURBOMACHINES
Exercises 7 - Industrial pumps

Pumped storage power plant

The Veytaux pumped storage power plant is operated by FMHL SA, Forces Motrices Hongrin-Léman, since 1970. The FMHL+ project doubled the capacity of the existing power plant by constructing Veytaux II, a new underground powerhouse with two ternary units. A cut-view of part of one of two hydrolectric units is given in Figure 1.



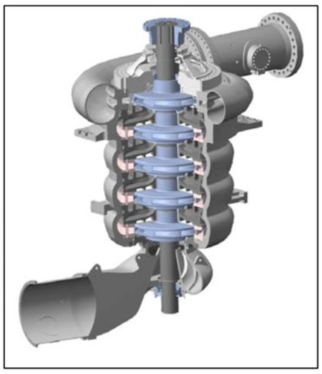


Figure 1 – Veytaux II, cut view of the multistage pump and part of the hydraulic circuit (left). 3D view of the multistage pump (right). Retrieved from www.alpiq.com

Pumping mode.

In Veytaux II, each multistage centrifugal pump installed in the two ternary units features the following specifications:

- Rotational speed $N = 600\ \text{min}^{-1}$
- Stage specific hydraulic energy $E_s = 1800\ \text{J}\cdot\text{kg}^{-1}$
- Total pump efficiency $\eta = 89\%$
- Bearing and disk friction losses: 1%
- Total volumetric losses: 2%
- Impeller inlet diameter $D_{\bar{1}e} = 1.32\ \text{m}$
- Shaft diameter $D_{shaft} = 0.68\ \text{m}$
- Tailwater level $Z_{\bar{B}} = 170\ \ \text{m}$

and the following physical conditions apply:

- Atmospheric pressure $p_a = 101325\ \text{Pa}$
- Water density $\rho = 998\ \text{kg}\cdot\text{m}^{-3}$
- Saturated vapor pressure $p_{vap} = 2343\ \text{Pa}$
- Gravity acceleration $g = 9.81\ \text{m}\cdot\text{s}^{-2}$.

1. How many stages can you recognize in the pump?

   This pump features 5 stages.

2. Several aspects/parameters are responsible of the onset of cavitation in a centrifugal pump in operation. Answer to the following questions regarding the cavitation phenomena inception:
   a. Which parameter set during the design phase of the storage pump installation (i.e. not modifiable during operation) should be carefully determined to reduce this risk?

   During the design phase, the Net Positive Suction Head (NPSH) determines how far we are from the risk of cavitation inception during operation depending on the setting level of the machine. A low NPSH means low pressure on the suction inlet, therefore a high risk of cavitation inception. The NPSH is defined as follows:

$$NPSH \approx \frac{p_a}{g\rho} - \frac{p_v}{g\rho} - h_s$$

   Therefore, the crucial parameter is the machine setting level $h_s = Z_{ref} - Z_{\bar{B}}$

   b. Which parameter that can be modified during operation has a strong impact on the cavitation onset? Is there an optimal operating condition regarding this parameter to minimize the risk of cavitation inception? If yes, what does this operating condition represent?

   The flow incidence, defined by the inlet relative flow angle and the blade pitch angle as $i = \beta_{\bar{1}e} - \beta_{b,\bar{1}e}$ *, affects the cavitation inception as well. When the machine operates at its Best Efficiency Point (BEP), the discharge Q is such that i=0 and the risk of cavitating is minimized.*

3. During lectures, we saw that not only the incidence angle $\beta_{\bar{1}e}$ has an influence, but the relative outlet flow angle β_{1e} has also an impact on the ideal condition of operation. Unfortunately, in real operation, this angle can never be optimal. Answer to the following questions:
   a. This deviation from the ideal scenario prevents the fulfilment of a particular condition: which one?

   Ideally, the no-slip condition would maximize the impeller efficiency.

   b. From a geometrical point of view, to which dimension do we refer to understand that β_{1e} does not fulfill this condition?

   The no-slip condition compares the outflow relative angle β_{1e} *to the blade angle at the trailing edge* $\beta_{b,1e}$*. The no-slip condition would be fulfilled in the case* $\beta_{1e} = \beta_{b,1e}$ *. However, in real operation, we have* $\beta_{1e} < \beta_{b,1e}$ *.*

   c. Which vector component of the outlet velocity triangle, lower compared to its ideal value because of this occurrence, describes the slip? Justify expressing the related equation.

   The tangential absolute velocity component of the outflow is lower than its ideal value, and it is calculated according to the following relation: $Cu_{1e,no-slip} - Cu_{1e} = (1-\mu)U_{1e}$ *, where* μ *is the slip factor (or deviation angle).*

4. Compute Z_{ref}, the setting elevation of the pump, to achieve a net positive suction head (NPSH) of 13.4 m.

   for a pump $NPSH = \dfrac{NPSE}{g}$ which yields $NPSH \approx \dfrac{p_a}{g\rho} - \dfrac{p_v}{g\rho} - h_s$

   $$Z_{ref} = \frac{p_a}{g\rho} - \frac{p_v}{g\rho} - NPSH + Z_{\bar{B}} = 166.6 \text{ m}$$

5. Compute the rated discharge value Q, assuming the stage unit specific speed is $n_{q,s} = N\dfrac{Q^{1/2}}{H_s^{3/4}} = 40.8$ (SI). Remember that unit factors are calculated with N in min^{-1}.

   $$H_s = \frac{E_s}{g} = 183.5 \text{ m}$$

   $$Q = \left(\frac{n_{q,s} H_s^{\frac{3}{4}}}{N}\right)^2 = 11.5 \text{ m}^3 \cdot \text{s}^{-1}$$

6. Compute the specific energy E of the pump and the hydraulic power. Then, using the correct efficiency term, deduce the input power P.

   $$E = z_s E_s = 9000.0 \text{ J} \cdot \text{kg}^{-1}$$

   $$P_h = \rho \cdot Q \cdot E = 103.23 \text{ MW}$$

   $$P = \frac{P_h}{\eta} = 116 \text{ MW}$$

7. Deduce the transformed specific energy $E_{t,s}$ for one stage.

   $$E_{t,s} = \frac{E_s}{\eta} \cdot \eta_m \cdot \eta_q = 1962.2 \text{ J} \cdot \text{kg}^{-1}$$

Let's now have a closer look to the water flux. For this operating condition, it is assumed that the inlet velocity $\vec{C}_{\bar{1}}$ is axial and uniformly distributed. The outlet flow is also radially uniform. All the flow distribution coefficients k_{cu} and k_{cm} of global Euler equation, defined as follows, are then assumed equal to 1.

$$E_{t,s} = k_{Cu_{1e}} U_{1e} Cu_{1e} - k_{Cu_{\bar{1}e}} U_{\bar{1}e} Cu_{\bar{1}e}$$

Moreover, the outlet velocity diagram corresponds to the maximum specific energy transfer.

8. Justify that in this case the global Euler equation reduces to $E_{t,s} = U_{1e} Cu_{1e}$.

   The inlet flow being axial therefore $Cu_{\bar{1}e} = 0$ *and assuming* $k_{cu1e} = 1$ *(radially uniform outflow) yield* $E_{t,s} = U_{1e} Cu_{1e}$

9. Express Cu_{1e} as a function of U_{1e}, Cm_{1e} and β_{1e}.

By trigonometry, $Cu_{1e} = U_{1e} - \frac{Cm_{1e}}{\tan\beta_{1e}}$

10. By mean of the expression computed in question 9) and the Euler equation, find an expression for the discharge to maximize the transferred power per stage, $P_{t,s}$.

Let's plug Cu_{1e} *computed in 9) in the Euler equation, and introduce the discharge by* $Cm_{1e} = \frac{Q_t}{A_1}$*;*

Then: $E_{t,s} = U_{1e}\left(U_{1e} - \frac{Cm_{1e}}{\tan\beta_{1e}}\right) = U_{1e}^{\ 2} - \frac{U_{1e}Q_t}{\tan\beta_{1e}A_1}$

Express the transformed power per stage as $P_{t,s} = \rho Q_t E_{t,s} = \rho Q_t\left(U_{1e}^{\ 2} - \frac{U_{1e}Q_t}{\tan\beta_{1e}A_1}\right)$

To maximize $P_{t,s}$ *:* $\frac{\partial P_{t,s}}{\partial Q} = 0 \rightarrow Q_t = \frac{U_{1e}\tan\beta_{1e}A_1}{2}$

11. Deduce that, for this optimal discharge condition, the relation $Cu_{1e} = \frac{U_{1e}}{2}$ holds.

Let's plug the optimal Q_t *in the previous expression of the transformed specific energy per stage:*

$$E_{t,s} = U_{1e}\left(U_{1e} - \frac{Cm_{1e}}{\tan\beta_{1e}}\right) = U_{1e}^{\ 2} - \frac{U_{1e}Q_t}{\tan\beta_{1e}A_1} = \frac{1}{2}U_{1e}^{\ 2}$$

And since $E_{t,s} = U_{1e}Cu_{1e}$*, we can say that for maximized power transfer,* $Cu_{1e} = \frac{U_{1e}}{2}$

12. Knowing the transformed specific energy per stage calculated in question 7), compute the impeller outlet diameter D_{1e}.

Knowing that Euler equation reduces to $E_{t,s} = Cu_{1e}U_{1e}$ *and that for the maximum power transfer* $Cu_{1e} = \frac{U_{1e}}{2}$ *therefore:*

$$U_{1e} = \sqrt{2E_{t,s}} = 62.6\ \mathrm{m\cdot s^{-1}}$$

$$\omega = \frac{2\pi\cdot N}{60} = 62.8\ \mathrm{rad\cdot s^{-1}}$$

$$D_{1e} = \frac{2U_{1e}}{\omega} = 1.99\ \mathrm{m}$$

13. Compute the meridional component at the impeller outlet Cm_{1e}. Consider the impeller channel height $B_1 = 0.17\ \mathrm{m}$.

$$A_1 = \pi D_1 B_1 = \ 1.06\ \mathrm{m}$$

$$Cm_{1e} = \frac{Q_t}{A_1} = \frac{Q}{\eta_q A_1} = 11.1\ \mathrm{m\cdot s^{-1}}$$

14. Compute the outlet absolute flow angle α_{1e}.

$$Cu_{1e} = \frac{U_{1e}}{2} = 31.3 \text{ m} \cdot \text{s}^{-1}$$

$$\alpha_{1e} = \tan^{-1}\left(\frac{Cm_{1e}}{Cu_{1e}}\right) = 19.53°$$

15. Compute the meridional component at the impeller inlet $Cm_{\bar{1}e}$.

$$A_{\bar{1}} = \frac{\pi\left(D_{\bar{1}e}^2 - D_{shaft}^2\right)}{4} = 1.005 \text{ m}^2$$

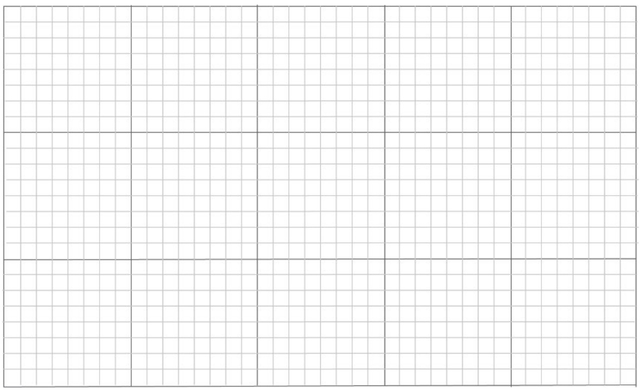
$$Cm_{\bar{1}e} = \frac{Q_t}{A_{\bar{1}}} = 11.6 \text{ m} \cdot \text{s}^{-1}$$

16. Compute the inlet relative inlet flow angle $\beta_{\bar{1}e}$.

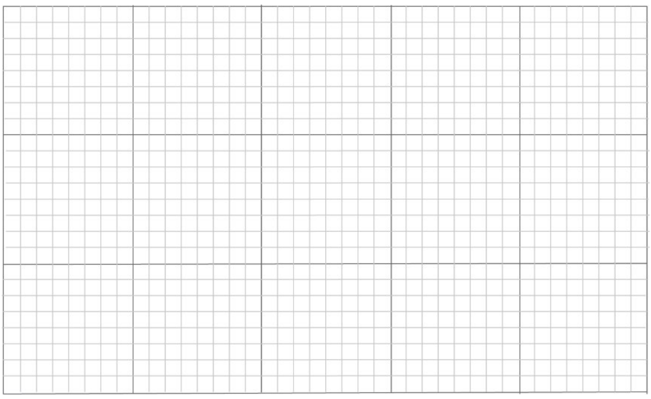
$$U_{\bar{1}e} = \omega \frac{D_{\bar{1}e}}{2} = 41.5 \text{ m} \cdot \text{s}^{-1}$$

$$\beta_{\bar{1}e} = \tan^{-1}\left(\frac{Cm_{\bar{1}e}}{U_{\bar{1}e}}\right) = 15.61°$$

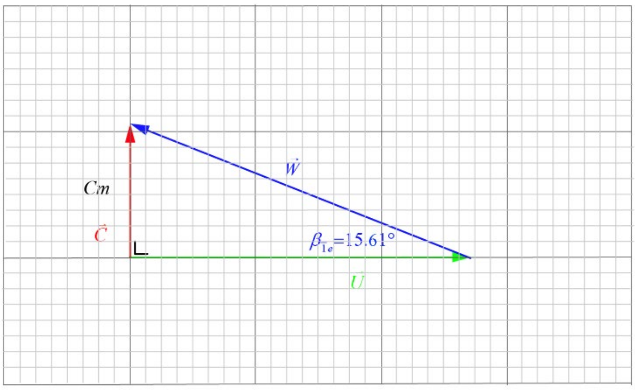
17. Sketch properly the velocity diagrams at the inlet and outlet of one impeller. Use the grid provided hereafter.



Impeller Inlet Velocity Diagram



Impeller Outlet Velocity Diagram



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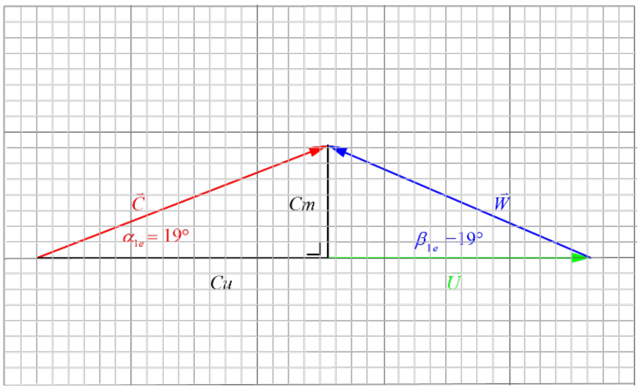

Impeller Outlet Velocity Diagram